University of Bahrain

College of Information Technology
Department of Computer Science

ITCS252 Discrete Structures
First Semester 2013/2014
Final Exam – 2 Hours

STUDENT NAME	
STUDENT#	
SECTION#	
SERIAL#	

This exam contains 7 pages (including this cover page) and 8 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to use Calculators.

You *are not allowed* to use books, notes, or mobiles

Question	Points	Score
1	10	
2	10	
3	9	
4	10	
5	8	
6	7	
7	8	
8	8	
Total:	70	

Instructor: Dr. Ali Alsaffar — Sections# 1 & 4
Dr. Yousif Al-Jazeeri — Sections# 2, 3 & 5 (Coordinator)

(1) (a) **[2 points]** What is the contrapositive of $p \to (q \to r)$

$\sim q \vee r$

$\sim(q \to r) \to \sim p = q \wedge \sim r \to \sim p$

$\sim p$ unless q $[\sim q \to p] = \sim(q \vee p) = \sim q \wedge \sim p$

(b) **[2 points]** What is the negation in English of "*Ahmed will not be happy unless he gets his degree*".

if Ahmde doesn't get his degree then hew will be happy

(c) **[2 points]** What is the converse of "*It is hot only if it is sunny*" in English. $q \to p$

it is summy only if it is hot

(d) **[2 points]** What are the values of $p, q, r,$ and s that make the statement $(p \wedge r \to \neg q) \to (q \to r \vee s)$ false.

T ∨ ? = T ;

T F

F → F

F →

Symbol	p	q	r	s
Value	T	T	F	F

\# (e) **[2 points]** Write any statement in the conclusion of the below conditional statement to make it a tautology.

$\sim p \vee p = p \to p$

$p \wedge \neg q \to$ $p \wedge \sim q$

$p \to q$

$= \sim p \vee q$

$= \sim(p \wedge \sim q) = \sim p \vee q$

$(\sim p \vee q)\ p$

(2) Let $e(x)$: "x is even",
$p(x)$: "x is prime",
$s(x)$: "x is a perfect square"

Write the following in symbolic form using only the quantifiers $e(x), p(x)$, and $s(x)$, where the domain is **Z**, the set of all integers.

(a) **[2 points]** Some integers that are not primes are not perfect squares.

$\exists x$: $\sim p(x) \wedge \sim s(x)$

(b) **[2 points]** Any perfect square is not a prime.

$\forall x$: $s x \to \sim p(x)$

(c) **[2 points]** A perfect square number is necessary for being odd. $s \to n$

$\exists x$: $\sim e(x) \wedge s(x)$

(d) **[2 points]** No perfect square is even.

$\forall x$: $s(x) \to \sim e(x)$

(e) **[2 points]** All even integers are neither perfect squares nor primes.

$\forall x$: $e(x) \to \sim s(x) \wedge \sim p(x)$

(3) [9 points] Using rules of inference show that the following argument is valid.

$p \rightarrow (q \rightarrow r)$ = $p \rightarrow (\sim q \vee r)$ = $\sim p \vee \sim q \vee r$
$p \vee s$
$t \rightarrow q$ = $\sim t \vee q$
$\neg s$
$\therefore \neg r \rightarrow \neg t$ = ~~$\sim r \vee$~~ $r \vee \sim t$

$\sim p \vee \sim q \vee r$
$p \vee s$
$\therefore \sim q \vee r \vee s$

$\sim q \vee r \vee s$
$\sim s$
$\therefore \sim q \vee r$

$\sim q \vee r = q \rightarrow r$
$t \rightarrow q$
$\therefore r \rightarrow t$

$p \vee s$
$\sim s$
$\therefore p$

$\sim p \vee \sim q \vee r$
p
$\therefore \sim q \vee r$ =
$\sim q \vee \sim t$
$\therefore r \vee \sim t = \sim r \rightarrow \sim t$

(4) **[10 points]** Prove by mathematical induction that $7^n - 2^n$ is divisible by 5, for any integer $n \geq 1$.

Basis : let $n=1$

$7^1 - 2^1 = 5 = 5 \cdot 1$

Inductive step : let $n=k$

Assume : $7^k - 2^k = 5w \rightarrow 7^k = 5w - 2^k$

Show $7^{k+1} - 2^{k+1} = 5z$

$7^{k+1} - 2^{k+1}$

$= 7 \cdot 7^k - 7 \cdot 2^k + 7 \cdot 2^k - 2 \cdot 2^k$

$7(7^k - 2^k) + 2^k(7-2)$

$7 \cdot 5w + 2 \cdot 5$

$5(7w + 2^k)$

(5) **[8 points]** For any real numbers a, b, and c. If $a+b+c=0$, prove that b^2-4ac is a perfect square. Use *Direct Proof.*

$\forall\, a,b,c \in R \quad a+b+c=0 \rightarrow b^2-4ac$ is a Pefect square

let $a+b+c=0$

To show $b^2-4ac=w^2$

$a+b+c=0$

$a+c=-b$

$(a+c)^2=b^2$

$a^2+2ac+c^2=b^2$

$b^2-4ac=a^2+2ac+c^2-4ac$

$=a^2-2ac+c^2$

$=(a-c)^2=w^2$

(6) **[7 points]** For any sets A and B, prove that $(A-B)\cup(A\cap B)\subseteq A.$

$(A-B)\cup(A\cap B)$

let $x\in A$

let $x\in A$ $(A\cap\bar{B})\cup(A\cap B)$

let $x\in A$ $A\cap(\bar{B}\cup B)$

let $x\in A$ $A\cap U$

let $x\in A$ A

(7) A relation R on $\mathbf{Z}$ is defined as

$$\forall a, b \in \mathbf{Z}: \; aRb \iff 3 | a^2 - b^2$$ $a^2 - b^2 = 3x$

(a) **[6 points]** Show that R is an equivalence relation.

① reflexive

$\forall s \in Z : aRa$

$aRa : a^2 - a^2 = 3x$

$0 = 3 \times 0$ T

② symmetric

$\forall a, b \in Z : aRb \to bRa$

assume $aRb \leftrightarrow a^2 - b^2 = 3x$

To show $b^2 - a^2 = 3w$

$a^2 - b^2 = 3x$

$-a^2 + b^2 = -3x$

$b^2 - a^2 = 3 \, -x$

$b^2 - a^2 = 3w$

④ transitive

(b) **[2 points]** Find three elements of the equivalence classes [0] and [-1].

$[0] =$

$[-1] =$

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Question	Points	Score
1	8	
2	8	
3	8	
4	4	
5	6	
6	6	
7	8	
8	4	
9	8	
Total:	60	

(1) **[8 points]** Let p:" *I will read the newspaper*" , q:" *I will buy the newspaper*" , r:" *I have money*" . Consider the statement:

" *if I have money, then I will buy the newspaper and read it*"

Answer the following questions for the above statement. $\sim(q \wedge p) = \sim q \vee \sim p$

(a) Give the Symbolic form.

$r \to q \wedge p$

(b) Give the contrapositive in English.

$\sim(q \wedge p) \to \sim r = \sim q \vee \sim r \to \sim r$

if i don't buy and ~~no~~ don't read then i don't have

(c) Give the negation in English. **Do not use "it is not the case".**

$\sim[r \to q \wedge p] = \sim[\sim r \vee (q \wedge p)] = r \wedge \sim(q \wedge p) = r \wedge \sim q \vee \sim p$

i don't have money and i will ~~need~~ buy or read

(d) Rewrite the original statement using *unless*. P unless q = $P \to \sim q$; $P \to \sim q \vee \sim p$

i will have money unless i don't buy ~~an~~ or i don't read

(2) **[8 points]** Given $P(x, y)$: " x plays with y " , $N(x)$: "x is nice " and $F(x)$ "x is funny ". Assume the domain for x and y is all students.

(a) Give the English for: $\exists x : \neg P(x, Jassim)$

~~Som~~

Some studen't don't play with Jassim

(b) Give the symbolic for : *All funny students play with all nice students.*

$\forall x : F(x) \to N(x)$

(c) Give the Symbolic for : *Not all students play with some nice students.*

$\sim \forall x : P(x, y) \to N(x)$; $\sim\forall x : P(x, y) \to$

~~$\exists(x) : P(x, y)$~~ $\forall x \; \exists x : \sim P(x, y) \wedge N(x)$

(d) Give the English for : $\neg\exists x, \forall y : F(y) \longrightarrow P(x, y)$. **Do not use "it is not the case".**

~~not some is funny~~

~~all studen't are~~

No studen't are funny plays with all studen'ts

(3) **[8 points]** The cat neither under the bed nor in the kitchen. The cat does not drink water unless it did not eat the cake. If the cat ate the mouse, then either it is under the bed or in the kitchen. The cat either drank water or ate the mouse. Therefore, if the cat ate the cake, then it is not under the bed.

(a) Convert the above propositions to symbols.

1- $\sim p \wedge \sim q$

2. $\sim r$ unless $\sim s$ = $s \rightarrow \sim r$ = $\sim s \vee \sim r$

3- $h \rightarrow p \vee q$ = $\sim h \vee p \vee q$

4- $r \vee h$

$\therefore s \rightarrow \sim p$ = $\sim s \vee \sim p$

(b) Show the above argument is valid.

L $\sim p \wedge \sim q$
$\therefore \sim p$
$\therefore \sim q$

$\sim h \vee p \vee q$
$\sim q$
$\therefore \sim h \vee p$

$\sim h \vee p$
$h \vee r$
$\therefore p \vee r$

$p \vee r$
$\sim s \vee \sim r$
$\therefore p \vee \sim s$

$\sim h \vee p \vee q$
$h \vee r$
$\therefore p \vee q \vee r$
$\sim q$
$\therefore p \vee r$ = $\sim p \rightarrow r$

$\sim s \vee \sim s$

$\sim p \rightarrow r$

(4) **[4 points]** True or False and give the reason.

(a) $\exists a \in Z, \forall b \in Z : a + b = 2.$

False $-1 + b = 2$

$a + b = 10$ is F for all No. $b \in Z$

(b) $\exists x \in R : \sqrt{\frac{x}{\sqrt{2}}}$ is rational but not integer.

True ; when $x = 3$ is rational

(c) $\forall p \in N$: if p is divisible by 5, then $2^p - 1$ is prime.

$P = \cancel{8k}\ 5k \rightarrow 2^p - 1$ is Prime

$T \rightarrow F = F$

(d) $\forall n \in Z^+, \exists t \in Q^+ : t^2 - n - 1 = 0.$

$t^2 = 1 + n$

True

(5) **[6 points]** Show that if n^2 is odd, then $n+1$ is even.

n^2 is odd $\rightarrow$ $n+1$ is even

~~n^2 is~~ $\sim q \rightarrow \sim p$ = $n+1$ is odd $\rightarrow$ n^2 is even

$n+1 = 2k+1$; $(2k+1)^2 = 4k^2 + 4k + 1 = 4$

$n^2 = 2k+1$ & $n = 2k+1$; $2k+1+1 = 2k+2 = 2(k+1)$

(6) **[6 points]** Let $T_0 = 0,\ T_n = 2T_{n-1} + 2^n$ for $n \geq 1$.

Show that $T_n = n \cdot 2^n$, for $n \geq 0$

- Base: let $P(0)$: $0 \cdot 2^0 = 0$ & T

- hypothesis: $P(k)$: $T_k = k \cdot 2^k$, $k \geq 0$

- Induction: $P(k+1)$: $T_{k+1} = k+1 \cdot 2^{k+1}$, $k \geq 0$

\# $T_n = 2\underline{T_{n-1}} + 2^n$ = ~~T_{k}~~ $2(k \cdot 2^k) + 2^{k+1}$

$= 2k \cdot 2^{k+1} + 2^{k+1} = 2^{k+1}(2k+1)$

(7) **[8 points]** (a) For any sets A, B, and C. Show that $\overline{(A \cup B \cup C)} \cup (A - (B \cup C)) = \overline{B \cup C}$

$\overline{A \cup B \cup C} \cup (A - (B \cup C) = (\bar{A} \cap \bar{B} \cap \bar{c}) \cup (A \cap (\overline{B \vee c}))$

$= (\bar{A} \cap \bar{B} \cap \bar{c}) \cup (A \cap (\bar{B} \wedge \bar{c})) = \bar{A} \cup \bar{A} \quad \cup$

$(\bar{B} \cap \bar{c}) \cup (\bar{A} \underset{\cup}{\cap} \bar{A}) = \bar{B} \cap \bar{c} = \overline{B \cup c}$

\# (b) Prove by contradiction, for any sets A and B , if $A \subseteq A - B$, then $A \cap B = \varnothing$

$A \subseteq A - B \rightarrow A \cap B = \varnothing$

suppose not : $A \subseteq A - B \wedge A \cap B \neq 0$

1. $A \subseteq (A \cap B^c) = \cancel{A \subseteq} \; x \in (A - B), \; x \in A \wedge x \notin B$

$\therefore x \in A \wedge x \in B^c$

$A \subseteq (A \cap B^c) = B^c \; : \; A \subseteq (A \cap \bar{B}) = A \subseteq A \cap A \subseteq \bar{B}$

$A \subseteq A - B \; ; \; x \in A \rightarrow x \in (A - B)$

$x \in A$

$\therefore x \in (A - B)$

$\therefore x \in A \wedge x \notin B$

$\therefore x \in A$

$\therefore x \notin B$

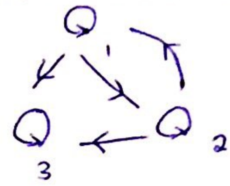
since $x \in A \wedge x \in B$

supposition false

$\therefore A \cap B \neq \varnothing$

(8) **[4 points]** Answer the following questions for each relation.

(a) Assume $R \subseteq A \times A$. Let $A = \{1, 2, 3\}$ and $R = \{(1,1), (1,2), (1,3), (2,1), (2,3)(2,2).(3.3)\}$. State the missing properties and explain why.



symetric ; (1,3 ∈ B ∧ (3,1) ∉ B

Antic :(1,2) ∈ R ; (2,1) but 2 ≠ 1

(b) Consider the relation at (a) : Is R an Equivalence Relation? Is R a Partial Order Relation? In each case state why.

~~transit~~ Not ; reflixe transitive but not summetric

Not

(9) **[8 points]** Given $R \subseteq Z^+ \times Z^+$. Let $aRb \iff ab \neq 0$. Prove or disprove relation properties.

① Reflixive : a.a ≠ 0 ; 1·1 ≠ 0

② symmetric : ~~aRb ⟶ bRA~~ if ab ≠ 0 then ~~ba ≠ 0~~ a ≠ 0 ∧ b ≠ 0

its symmetric

③ anti symmetric : ab ≠ 0 ∧ ba ≠ 0 ; 1*2 ≠ 0 ∧ 2*1 ≠ 0

but 2 ≠ 2

④ transitive : (xRy) ∧ (yRz) ⟶ x R z

ab ≠ 0 ∧ bc ≠ 0 ⟶ ac .

1*2 ∧ 2*3 ≠ 0 1*3 ≠ 0

2

∴ Transitive

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mobiles, books and notes are not allowed. Calculator is allowed. Please show your work for each question

Question	Points	Score
1	6	
2	6	
3	8	
4	6	
5	6	
6	6	
7	6	
8	6	
9	8	
10	6	
Total:	64	

(1) **[6 points]** Given:
p: "The cat will bite mouse" , q: "The cat will play with the mouse "
r: "The cat will be happy" , s: "The mouse will be happy"

Answer the following:

(a) Write the english statement using *whenever* : $\neg p \longrightarrow r \wedge s$

(b) Write the symbolic for : The mouse will be happy unless both the cat will not be happy and will bite the mouse .

(c) Write in English using only (*or* , *not*) : $p \wedge \neg q \longrightarrow \neg r$

(2) **[6 points]** Let : $E(x)$:"x is even number", $P(x)$: "x is prime", $V(x, y)$: "x is divisible by y". Where the domain for x and y is Z . Answer the following questions.

(a) Write the English statement for : $\forall x \in Z, \forall y \in Z : E(x) \longrightarrow \neg V(x, y)$

(b) Write a symbolic statement for: *All prime integers are odd.* .

(c) Write english statement for: $\exists x \in Z, \exists y \in Z : V(x, y) \wedge \neg E(y)$.

(3) **[8 points]** Hellen was not in the bedroom and the light was on. The light was off when the diamond ring was stolen. If Hellen went to the bedroom, then the light was off. The diamond ring is stolen or Hellen was in kitchen . If Hellen was in the kitchen, then She drank water. If hellen ate an orange then she was thirsty and did not drink water. Therefore, neither the diamond ring stolen nor Hellen ate an orange .

(a) Convert the above argument into symbolic form.

(b) Using the Inference rules, to show the above argument is valid.

(4) **[6 points]** True or False. Justify your answer.

(a) $\exists x, y \in Z : \frac{x}{y} = x + y$.

$\frac{1}{0} = 1 + 0$

$x = -4 \quad y = 2$

T. $\frac{-4}{2} = -2 \quad , -4 + 2 = -2$

(b) $\forall x, y \in Z : x^2 + y^2 > 0.$ $\geq$ لوكانت بحر T

T. sinse ~~x, y ∈ Z~~ $x^2 \geq 0$ and $y^2 \geq 0$

$\cancel{x}^2 + y^2 \geq 0$

(c) $\exists x \in Z, \forall y \in R : y^2\sqrt{x} = 0.$

True $x = 0$

$y^2 \cdot \sqrt{0} = y^2 \cdot 0 = 0$

$\forall y \in R \; \exists x \in Z \cdot y^2 \sqrt{x} = 0$

$\sqrt{x} = \frac{0}{y} = 0 \rightarrow x = 0$

true

(5) **[6 points]** Show that $n(n^2 - 1)(n + 2)$ is divisible by 4 for all integer n.

Case1 n is even $n = 2K$

Case1: $n(n^2 - 1)(n + 2) = 2K(4K^2 - 1)(2K + 2)$

$= 2 \cdot 2[K(4K^2 - 1) \cdot (K + 1)]$

$= 4 \cdot w \quad w \in R$

$\therefore n \cdots$ divisbal by 4

Case2: n is odd $n = K + 1$

$n(n^2 - 1) \cdot (n + 2)$

$(2K + 1)((2K + 1)^2 - 1) \cdot (2K + 1 + 2)$

$(2K + 1)(4K^2 + 4K + 1 - 1) \cdot (2K + 3)$

$(2K + 1)(4K^2 + 4K \cancel{+1-1})(2K + 3)$

$4 \cdot [(2K + 1) \cdot (K^2 + K) \cdot (2K + 3)]$

$4 \cdot w$

(6) **[6 points]** Show using induction that $\frac{2}{3} + \frac{2}{3^2} + ... + \frac{2}{3^n} = 1 - (\frac{1}{3})^n$. for $n \geq 1$

لازم اكتب ذي

جزئ

Bais مو

① Basis let $n=1$

$LHS = \frac{2}{3} = \frac{2}{3}$

$RHS = 1-(\frac{1}{3})^1 = 1-\frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

① Inductive step : let $n=k$

لازم اكتبه → suppose $\frac{2}{3} + \frac{2}{3} + \cdots \frac{2}{3^k} = 1 = (\frac{1}{3})^k$

we want to show

$\frac{2}{3} + \frac{2}{3^2} + \cdots + \frac{2}{3^k} = 1 - (\frac{1}{3})^{k+1}$

$\frac{2}{3} + \frac{2}{3^2} + \cdots + \frac{2}{3^k} + \frac{2}{3^{k+1}}$

$1 - (\frac{1}{3})^k + \frac{2}{3k+1}$

$1 - (\frac{1}{3})^{k+1-1} + 2 \cdot (\frac{1}{3})^{k+1}$

مثلا

$= 3x + 2x$

$1 - (\frac{1}{3})^{-1} \cdot (\frac{1}{3})^{k+1} + 2(\frac{1}{3})^{k+1}$

$1 - 3(\frac{1}{3})^{k+1} + 2(\frac{1}{3})^{k+1}$

$1 - (\frac{1}{3})^{k+}$

$\frac{1}{x^n} = x^{-n}$

$\frac{1}{x^n} +$

$\frac{1^n}{x^n} = (\frac{1}{x})^n$

$\frac{1}{3^{k+1}} = \frac{1^{k+1}}{3^{k+1}}$

$= (\frac{1}{3})^{k+1}$

(7) **[6 points]** Show using set identities that $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C}) = \overline{A} \cup \overline{B} \cup \overline{C}$

let $S = \bar{A} \cup \bar{B}$ $T, A \cap B$

$S \cup (T \cap \bar{C})$

$(S \cup T) \cap (S \cup \bar{C})$

$[(\bar{A} \cup \bar{B}) \cup (A \cap B)] \cap [(\bar{A} \cup \bar{B}) \cup \bar{C}]$

$[\bar{A} \cup \bar{B} \cup A] \cap [\bar{A} \cup \bar{B} \cup B] \cap [\bar{A} \cup \bar{B} \cup \bar{C}]$

$[\bar{A} \cup A \cup \bar{B}] \cap [\bar{A} \cup \bar{B} \cup B] \cap [\bar{A} \cup \bar{B} \cup \bar{C}]$

$U \cup B$ $\cap$ $U \cup \bar{A}$ $\cap$ "

U "

= "

(8) **[6 points]** Prove by contradiction that if $A \subseteq B$, then $A - B = \varnothing$

If $A \subseteq B$, then $A - B = \varnothing$

by contradiction

suppose there is 2 sets A, B such that $A \subseteq B$ and $B - B \neq \varnothing$

let $x \in A - B$

$x \in A \wedge x \in B$

$\therefore x \in A$ / $\therefore x \in B$

Since $A \subseteq B$: $x \in A \to x \in B$

$x \in A$

$\therefore x \in B$

$x \in B$

$\therefore x \in B \wedge x \in B$

$A - B = \varnothing$

(9) **[8 points]** Answer the following questions: Given $A = \{1, 2, 3\}$ .Let R be relation on A such that $R = \{(1,1), (2,2), (1,2)\}$. Is R reflexive, symmetric, antisymmetric. transitive? Explain .

$\overset{a\ b}{(1,2)}$ $(2,2)$

① not Reflexive : $(3,3) \notin R$

② not symatric : $(1,2) \in R$ but $(2,1) \notin R$

③ Antisymatric

$(1,2) \in R \land (2,1) \in R \rightarrow 1=2$

$T \land F \rightarrow F = T$

$(1,1) \in R \land (1,1) \in R \rightarrow 1=1\ T$

$(2,2) \in R \land (2,2) \in R \rightarrow 2=2\ T$

④ transitive: $(1,2) \in R \land (2,2) \in R \rightarrow (1,2) \in R\ T$

$(1,1) \in R \land (1,2) \in R \rightarrow (1,2) \in R\ T$

$(1,1) \in R \land (1,1) \in R \rightarrow (1,1) \in R\ T$

$(2,2) \in R \land (2,2) \in R \rightarrow (2,2) \in R\ T$

(10) [6 points] $\forall a, b \in Z : aSb \iff 3a + 5b$ is divisible by 8. . Is S equivalenc relation? prove or disprove.

① reflecsive

$\forall a \in Z : aSa$

$aSa \iff 3a + 5a$ is divisble by 8

$3a + 5a = 8a$ T

② symmetric

$\forall a, b \in Z : aSb \to bSa$

let $aSb \to 3a + 5b = 8x$

To show $bSa \to 3b + 5a = 8w$

$3a + 5b = 8x$

$-3a - 5b = -8x$

$8a - 3a + 8b - 5b = -8x$

$5a + 3b = 8(a + b - x)$

③ transitive $\forall a, b, c \in Z . aSb \wedge bSc \to aSc$

let $aSb \wedge bSc$

To show aSc $3a + 5c = 8y$

$3a + 5h = 8x$

$3b + 5c = 8w$

$8b + 5c = 8x + 8w$

$3a + 5c = 8(x + w - b)$